DISSERTATIO ASTRONOMICA,

DE

CREPUSCULIS.

QUAM

CONS. AMPL. FAC. PHIL. IN ACAD. ABOËNS.

PRÆSIDE

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ABOÆ, TYPIS FRENCKELLIANIS.

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§. 1.

Atmosphæram terrestrem, radios Solares reflectendo, creperam illam atque debilem lucem mane ante ortum solis & vesperi post occasum ipsius perspicuam, quam crepusculum nominamus, producere, satis est cognitum. Postquam videlicet Tellus motu diurno nos e conspectu Solis subduxit, sublimior aër nobis adhuc a Sole illustratus manet. Sed magis magisque descendente Sole, minus continuo illustratur aër. Pariter mane Sol paullatim Atmosphæram illuminare incipit, coelique faciem undique minutatim lucidam usque ad ortum ipsius reddit. Successive itaque crescit & decrescit crepusculum, & a qualitate aëris, ejusque vicissitudinibus pendet; quamobrem momentum initii atque finis ipsius difficile admodum est determinatu. Quum autem ex observationibus liqueat, Stellas minores oculo inarmato cerni posse eo momento, quo Sol decimum octavum infra horizontem attigerit gradum, Astronomi assumserunt, limites crepusculorum ita esse determinandos, ut portio arcus diurni, quam Sol a momento ortus vel occasus absolverit, donec ad almicantarat *18°* infra horizontem descenderit, crepusculum metiatur. Unde patet, durationem ipsius & a motu Solis in ecliptica & a diversa locorum Latitudine quoque & quidem maxime pendere. Quo enim propius Solis supra horizontem ortus, vel infra ipsum occasus, ad motum horizonti perpendicularem accesserit, eo citius Sol ascendat descendatve, necesse est, unde crepusculum brevius: quo autem obliquior motus Solis orientis vel occidentis versus horizontem fuerit, eo lentiores Solis ab horizonte distantia mutationes subibit, unde crepusculum longius evadit. Variante igitur declinatione Solis atque elevatione Poli, variat quoque crepusculum: specimen ideo Academicum edituri, durationem crepusculi nobis ita investigandam proposuimus, ut tradita generali Problematis solutione, specialem ipsius applicationem, assumta Latitudine 60.° 27'. 7", pro quovis dimidio declinationis gradu exhibeamus, unde crepusculum pro quovis die facili negotio haberi potest.

§. 2.

Est vero directa solutio Problematis nostri inventu haud difficilis. Sit nempe *ZP* arcus meridiani,

diani, Z Zenith, *P* Polus, *HR* horizon verus, si locus Solis in horizonte fuerit *A*, jungantur puncta *A* & *Z* arcu *AZ*, ductoque arcu circuli maximi *ZB* ita, ut æqualis fiat 108°, & productus secet arcum diurnum *DB* per *A* transientem in *B*, junctisque punctis *A* & *P*, *B* & *P*, arcubus circulorum maximorum *AP* & *BP* habebitur in ΔZAP, posito Sinu toto = 1 (Element. Trigon. Sphær.)

$$Sin\,\tfrac{1}{2} ZPA = \sqrt{\frac{Sin\,\tfrac{1}{2}(AZ+AP-ZP)\,Sin\,\tfrac{1}{2}(AZ+ZP-AP)}{Sin\,ZP\;Sin\,AP}}$$

& in ΔZPB

$$Sin\,\tfrac{1}{2} ZPB = \sqrt{\frac{Sin\,\tfrac{1}{2}(BZ+BP-ZP)\,Sin\,\tfrac{1}{2}(BZ+ZP-BP)}{Sin\,ZP\;Sin\,BP.}}$$

Ex quibus itaque innotescunt anguli *ZPA* & *ZPB*, adeoque etiam $> APB = ZPB - ZPA$, qui quidem in tempus conversus, dabit crepusculum quæsitum.

Exempl. I. In Latitudine 60°. 27'. 7". si pro die 3 Aprilis anni currentis declinatio Solis Borealis in momento occasus fuerit 5°. 2'. 17". crepusculum vespertinum pro eodem die sequenti calculo investigatur:

$ZP = 29.° 32'. 53''$ $\quad$ $Log\,Sin\,\frac{AZ+AP-ZP}{2} = \bar{1},9799110$

$AP = PB = 84.° 57' 43''$ $\quad$ $Log\,Sin\,\frac{AZ+ZP-AP}{2} = \bar{1},4731352$

$AZ = 90°$ $\quad$ $-\,Log\,Sin\,ZP = 0,3070180$

$BZ = 108°$ $\quad$ $-\,Log\,Sin\,AP = 0,0016812$

$2\,Log\,Sin\,\tfrac{1}{2}\,ZPA = \bar{1},7617454$

$\frac{AZ+AP-ZP}{2}=72^\circ.42'.25''$. $Log\,\frac{1}{2}Sin\,ZPA=\bar{1},8008727$

$\frac{AZ+ZP-AP}{2}=17^\circ.17'.35''$

$\frac{BZ+BP-ZP}{2}=81^\circ.42'.25''$

$\frac{BZ+ZP-BP}{2}=26^\circ.17'.35''$

$\frac{1}{2}ZPA=49^\circ.28'.24''$

$ZPA=98^\circ.56'.48''$

$\frac{1}{2}ZPB=70^\circ.50'.24'',94$, $Log\,Sin\,\frac{BZ+BP-ZP}{2}=\bar{1},9954349$

$ZPB=141^\circ.40'.49''.88$, $Log\,Sin\,\frac{BZ+ZP-BP}{2}=\bar{1},6463670$

$-\,Log\,Sin\,ZP=0,3070180$

$-\,Log\,Sin\,BP=0,0016812$

$2\,Log\,Sin\,\frac{1}{2}ZPB=\bar{1},9505011$

$Log\,Sin\,\frac{1}{2}ZPB=\bar{1},9752505$

Datis vero jam angulis ZPA & ZPB, ipſorum quoque dabitur differentia ſeu angulus $APB=42^\circ.44'.1'',88$, qui in tempus converſus ſecundum rationem $360^\circ:24^h$, exhibet crepuſculum veſpertinum quæſitum $2^h.50'.56'',12$.

Exempl. 2. Eodem loco & anno exſiſtente declinatione Solis Auſtrali in momento ortus ipſius die 7 Octobris $5^\circ.14'.32''$, crepuſculum matutinum ſic computatur:

$ZP=$

$ZP = 29^\circ.\ 32'.\ 53''$ $\quad \text{Log Sin}\ \frac{AZ + AP - ZP}{2} = \bar{1},9901566$

$AP = PB = 95^\circ.14'.32''$ $\quad \text{Log Sin}\ \frac{AZ + ZP - AP}{2} = \bar{1},3232965$

$AZ = 90^\circ$ $\quad - \text{Log Sin}\ ZP = 0,3070180$

$BZ = 108^\circ$ $\quad - \text{Log Sin}\ AP = 0,0018203$

$\frac{AZ + AP - ZP}{2} = 77^\circ.50'.49'',5,$ $\quad 2\,\text{Log Sin}\ \tfrac{1}{2} ZPA = \bar{1},6222914$

$\frac{AZ + ZP - AP}{2} = 12^\circ.9'\,10'',5$ $\quad \text{Log Sin}\ \tfrac{1}{2} ZPA = \bar{1},8111457$

$\frac{BZ + BP - ZP}{2} = 86^\circ.50'.49'',5$

$\frac{BZ + ZP -- BP}{2} = 21^\circ.9'.10'',5$

$\tfrac{1}{2} ZPA = 40^\circ.20'.34'',1$ $\quad \text{Log Sin}\ \frac{BZ + BP -- ZP}{2} = \bar{1},9993422$

$ZPA = 80^\circ.41'.8'',2$ $\quad \text{Log Sin}\ \frac{BZ + ZP - BP}{2} = \bar{1},5573357$

$\tfrac{1}{2} ZPB = 58^\circ.55'.57''$ $\quad - \text{Log Sin}\ ZP = 0,3070180$

$ZPB = 117^\circ.51'.54''$ $\quad - \text{Log Sin}\ BP = 0,0018203$

$2\,\text{Log Sin}\ \tfrac{1}{2} ZPB = \bar{1},8655162$

$\text{Log Sin}\ \frac{ZPB}{2} = \bar{1},9327581$

Unde itaque innotescit $> APB = 37^\circ.10'.45''8$, qui eodem modo, quo in exemplo antecedenti in tempus conversus dabit crepusculum matutinum $2^h.\ 28'.\ 43''$.

§ 3.

§. 3.

Quamvis crepuſculum ſecundum methodum in §. præcedenti allatam inveſtigari poſſit, ope tamen formularum Trigonometricarum algebraica ipſius inveſtigatio multo eſt concinnior. Manente igitur conſtructione figuræ eadem, ſi declinatio Solis fuerit d, Latitudo Loci p, arcus $BK = 18^\circ = a$, $> ZPA = \gamma$ & $> APB = y$, erit $AP = PB = 90^\circ \pm d$, $ZP = 90^\circ - p$, $ZA = 90^\circ$, $ZB = 90^\circ + a$ & $> ZPB = \gamma + y$. In triangulo vero ZPA habebitur (Elem. Trig. Sphær.) $Cos\,AZ = Cos\,ZPA\,Sin\,ZP\,Sin\,AP + Cos\,ZP\,Cos\,AP$, & in ΔZPB, $Cos\,BZ = Cos\,ZPB\,Sin\,ZP\,Sin\,BP + Cos\,ZP\,Cos\,BP$, ſeu $Cos\gamma\,Cos p\,Cos d \pm Sin\,p\,Sin\,d = 0$ (I) & $Cos\,\overline{90^\circ + a} = Cos\,\overline{\gamma + y}\,Cos p\,Cos d \pm Sin\,p\,Sin\,d$ (II). Subducendo vero æquationem (I) ab æqu. (II) eruitur $Cos\,\overline{90^\circ + a} = (Cos\,\overline{\gamma + y} - Cos\gamma)\,Cos p\,Cos d$ & facta debita reductione $-Sin\,a = (Cos\,\overline{\gamma + y} - Cos\gamma)\,Cos p\,Cos d$, unde $Cos(\gamma + y) = Cos\gamma - \frac{Sin\,a}{Cos p\,Cos d}$ (III). Quumque dividendo æquat. (I) per $Cos p\,Cos d$, ſit $Cos\gamma = \mp tg\,p\,tg\,d$, valor ipſius γ eſt datus & hinc facillime determinatur $\gamma + y$ & y ope æqu. (III), qui crepuſculum exhibet. Quo autem angulus iſte Logarithmorum ope ex æquat. (III) inveniri queat, ſtatuatur in caſu, quo $Cos\gamma$ poſitivus fuerit, atque $Cos\gamma > \frac{Sin\ a}{Cos p\ Cos d}$ vel $Sin d < \frac{Sin\ a}{Sin\ p}$, quo in caſu $\gamma + y$

$< 90^\circ$,

$< 90^\circ$, $\frac{Sin\,a}{Cos\gamma\, Cos p\, Cos d} = Sin\,\varphi^2$, unde facta substitutione $Cos(\gamma + y) = Cos\gamma\, Cos\varphi^2$. Si vero fuerit $Cos\gamma = \frac{Sin\,a}{Cos p\, Cos d}$, vel $Sin\,d = \frac{Sin\,a}{Sin\,p}$, erit $\gamma + y = 90^\circ$, e qua æquatione, cognito γ, y facillime determinatur. Quum autem $Cos\gamma < \frac{Sin\,a}{Cos p\, Cos d}$, vel $Sin\,d > \frac{Sin\,a}{Sin\,p}$ & $\gamma + y > 90^\circ$, ponatur $\frac{Cos\gamma\, Cos p\, Cos d}{Sin\,a} = Sin\,\varphi^2$, eritque $-Cos(\gamma + y) = \frac{Cos\,\varphi^2\, Sin\,a}{Cos p\, Cos d}$. Exsistente denique $Cos\gamma$ negativo, supponatur $\frac{Sin\,a}{Cos\gamma\, Cos p\, Cos d} = tg\,\varphi^2$, eruiturque facta debita reductione $Cos\,\overline{\gamma + y} = -\frac{Cos\gamma}{Cos\,\varphi_2}$.

Exempl. Si resumantur eadem data, quæ in exemplo 1. §. præc. attulimus, crepusculum secundum methodum jam allatam sequenti modo investigatur.

$d = 5^\circ.\ 2'.\ 17''$	$Log\ tg\ p = 0{,}2465085$
$p = 60^\circ.\ 27'.\ 7''$	$Log\ tg\ d = \bar{2}{,}9452616$
$a = 18^\circ$	$Log\ Cos\gamma = \bar{1}{,}1917701$
$\gamma = 98^\circ.\ 56'.\ 48''$,	$Log\ Sin\ a = \bar{1}{,}4899824$
$\gamma + y = 141^\circ.\ 40'\ 50''$	$-\ Log\ Cos p = 0{,}3070180$
$y = 42^\circ.\ 44'.\ 2''$	$-\ Log\ Cos\ d = 0{,}0016811$

— Log

$$-\text{Log Cos}\,\gamma = 0{,}8082299$$
$$\text{Log tg}\,\varphi^2 = \bar{0}{,}6069114$$
$$\text{Log Cos}\,\gamma = \bar{1}{,}1917701$$
$$-\text{Log Cos}\,\varphi^2 = 0{,}7028566$$
$$\text{Log Cos}(\gamma + y) = \bar{1}{,}8946267$$

Dato itaque y, crepusculum eodem modo quo supra indicavimus habetur.

Schol. 1. In casu quo Sol 18 gradus infra horizontem non percurrit, vel si ipsius locus exacte $18°$ fuerit, crepusculum per totam noctem durare, perspicuum est. Accidit vero hoc in Latitudine $60° 27' 7''$, ubi declinatio Solis Borealis $11°. 32'. 53''$ excesserit. In tabula igitur apposita, valores γ & y continente, crepusculum tantummodo pro declinatione Solis Bor. $11° 30'$, quam circa diem 21 mensis Aprilis obtinet, calculavimus. Nam ab eo tempore crescit & decrescit declinatio Solis Borealis successive, ita tamen ut semper sit major $11°. 30'$, donec circa diem 23 mensis Augusti hanc ipsam iterum habet magnitudinem. Diminuitur vero posthac declinatio Solis Borealis eodem modo, quo antea crevit, quamobrem durationes crepusculorum eædem sunt.

Declinatio Solis Bor.	γ	y	Declinatio Solis Austr.	γ	y	Declinatio Solis Austr.	γ	y
0°.	6h. 0'. 0"	2h. 35'. 12"	0°.	6h. 0'. 0"	2h. 35'. 12"	12°.	4h. 30'. 55"	2h. 29'. 43"
0° 30'	6h. 3'. 32"	2h. 36'. 15"	0°. 30'	5h. 56'. 28"	2h. 34'. 15"	12°. 30'	4h. 27'. 55"	2h. 30'. 12"
1°.	6h. 7'. 3"	2h. 37'. 23"	1°.	5h. 52'. 57"	2h. 33'. 22"	13°.	4h. 23'. 52"	2h. 30'. 43"
1°. 30'	6h. 10'. 35"	2h. 38'. 37"	1°. 30'	5h. 49'. 25"	2h. 32'. 34"	13° 30'	4h. 19'. 46"	2h. 31'. 17"
2°.	6h. 14'. 8"	2h. 39'. 57"	2°.	5h. 45'. 52"	2h. 31'. 51"	14°.	4h. 15'. 38"	2h. 31'. 55"
2°. 30'	6h. 17'. 40"	2h. 41'. 24"	2°. 30'	5h. 42'. 20"	2h. 31'. 11"	14°. 30'	4h. 11'. 26"	2h. 32'. 37"
3°.	6h. 21'. 13"	2h. 42'. 59"	3°.	5h. 38'. 47"	2h. 30'. 36"	15°.	4h. 7'. 10"	2h. 33'. 23"
3°. 30'	6h. 24'. 46"	2h. 44'. 41"	3°. 30'	5h. 35'. 14"	2h. 30'. 5"	15°. 30'	4h. 2'. 51"	2h. 34'. 14"
4°.	6h. 28'. 20"	2h. 46'. 32"	4°.	5h. 31'. 40"	2h. 29'. 37"	16°.	3h. 58'. 27"	2h. 35'. 8"
4°. 30'	6h. 31'. 55"	2h. 48'. 3"	4°. 30'	5h. 28'. 5"	2h. 29'. 14"	16°. 30'	3h. 53'. 59"	2h. 36'. 7"
5°.	6h. 35'. 31"	2h. 50'. 45"	5°.	5h. 24'. 29"	2h. 28'. 52"	17°.	3h. 49'. 27"	2h. 37'. 11"
5°. 30'	6h. 39'. 7"	2h. 53'. 10"	5°. 30'	5h. 20'. 53"	2h. 28'. 34"	17°. 30'	3h. 44'. 50"	2h. 38'. 19"
6°.	6h. 42'. 44"	2h. 55'. 48"	6°.	5h. 17'. 16"	2h. 28'. 21"	18°.	3h. 40'. 7"	2h. 39'. 33"
6°. 30'	6h. 46'. 23"	2h. 58'. 42"	6°. 30'	5h. 13'. 37"	2h. 28'. 12"	18° 30'	3h. 35'. 18"	2h. 40'. 52"
7°.	6h. 50'. 2"	3h. 1'. 55"	7°.	5h. 9'. 58"	2h. 28'. 3"	19°.	3h. 30'. 24"	2h. 42'. 17"
7°. 30'	6h. 53'. 43"	3h. 5'. 29"	7°. 30'	5h. 6'. 17"	2h. 27'. 58"	19° 30'	3h. 25'. 21"	2h. 43'. 49"
8°.	6h. 57'. 25"	3h. 9'. 29"	8°.	5h. 2'. 35"	2h. 27'. 58"	0°.	3h. 20'. 13"	2h. 45'. 28"
8°. 30'	7h. 1'. 5"	3h. 14'. 2"	8°. 30'	4h. 58'. 55"	2h. 28'. 0"	20°. 30'	3h. 14'. 56"	2h. 47'. 13"
9°.	7h. 4'. 54"	3h. 19'. 15"	9°.	4h. 55'. 6"	2h. 28'. 5"	21°.	3h. 9'. 31"	2h. 49'. 7"
9°. 30'	7h. 8'. 41"	3h. 25'. 23"	9° 30'	4h. 51'. 19"	2h. 28'. 13"	21°. 30'	3h. 3'. 57"	2h. 51'. 10"
10°.	7h. 12'. 29"	3h. 32'. 48"	10°.	4h. 47'. 31"	2h. 28'. 25"	22°.	2h. 58'. 10"	2h. 53'. 22"
10°. 30'	7h. 16'. 20	3h. 42'. 11"	10°. 30'	4h. 43'. 40"	2h. 28'. 39"	22° 30'	2h. 52'. 14"	2h. 55'. 45"
11°.	7h. 20'. 13	3h. 55'. 19"	11°.	4h. 39'. 47"	2h. 28'. 57"	23°.	2h. 46'. 7"	2h. 58'. 16"
11°. 30'	7h. 24'. 8"	4h. 22'. 42"	11°. 30'	4h. 35'. 52"	2h. 29'. 18"	23°. 28	2h. 40'. 5"	3h. 0'. 56"

D

Schol.

Schol. 2. Ut vero pateat usus tabulæ allatæ, quæratur exempli gratia crepusculum vespertinum pro die 9 Mensis Novembris hujus Anni, exsistente declinatione Solis Australi in momento occasus *16°.44′.56″*. In tabula habetur crepusculum pro decl. 16°.30′, 2h.36′.7″ & pro decl. 17°, 2h.37′ 11″: Inferendo igitur 30′ (differentia inter declinationes in tabula): *14′,56″* (differentia inter *16°.30′* & *16°.44′.56″*): *1′.4″* differentia inter crepuscula pro *16°. 30′* & *17°*): *31″, 9* (differentia inter crepuscula pro *16° 30′* & *16°. 44′.56″*), si addatur, quoniam e tabula crepuscula crescere animadvertimus, *31″, 9*, crepusculo *2h.36′.7″* pro *16°.30′*, eruitur *2h.36′. 38″9*, seu crepusculum vespertinum quæsitum. Si vero decrescens fuerit crepusculum, differentia inventa auferenda est.

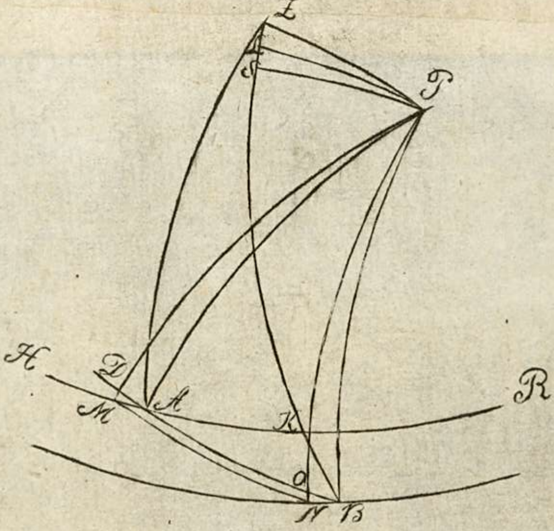
Schol. 3. Valorem anguli y, arcum semidiurnum exhibentem attulimus, quoniam crepusculum illo incognito investigari nequit, & præterea ortus Solis atque occasus ejus ope facillime determinatur. Si videlicet momento culminationis Solis addatur, exhibet occasum, sin autem auferatur, ortum. Necesse tamen est, ut ex Ephemeridibus depromatur declinatio Solis pro occasu vel ortu, & e tabula, eodem plane modo, quo Schol. 2. indicavimus, investigetur angulus y huic momento respondens, qui denique ope formularum pro invenienda refractione Astronomica corrigatur.

§. 4.

§. 4.

Quod ad problema de inveniendo crepusculo minimo attinet, sequens illud resolvendi simplicissima nobis videtur methodus. Demonstrandum enim primo est, angulos positionis ZAP & ZBP vel ut a quibusdam nuncupantur parallacticos, in casu, quo crepusculum est minimum, æquales esse, & deinde quærenda est declinatio Solis, exsistentibus his angulis æqualibus. Concipiatur igitur arcus MN infinite proximus ipsi AB, atque sit NB portio arcus horizonti paralleli, erit, exsistente $>APB$ minimo, $AB=MN$. Quumque præterea quam proxime æquales sunt arcus Do & MN, atque MN ipsi AB parallelus, sequitur, ut assumi possit $AD=oB$ & $AM=NB$. Æqualia igitur sunt triangula DMA & BoN, adeoque etiam angulus $DAM=$ang. oBN. Est autem $>ZAH=90^\circ=>PAD$, unde, sublato communi $>ZAD$, habebitur $>PAZ=>DAM$. Pariter angulus $ZBN=90^\circ=>PBA$, quamobrem, si communis $>ZBA$ auferatur, erit $>PBZ=>oBN$, atque hinc sequitur, ut sit $>PAZ=>PBZ$, in casu quo $>APB$ minimus est. Sumatur præterea $SB=90^\circ$, erit $ZS=BK=18^\circ$, atque $\Delta AZP=\Delta PSB$, unde $ZP=PS$ & demisso arcu PL normali ipsi ZB, habebitur $ZL=SL=9^\circ$. In ΔLPB rectangulo erit $Cos\,PB=Cos\,LB\;Cos\,LP$ & in ΔLPS etiam rectangulo $Cos\,PS=Cos\,LP\;Cos\,LS$, unde eruitur

Cos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$\frac{Cof\,BP}{Cof\,LB} = \frac{Cof\,SP}{Cof\,LS}$, atque hinc $Cof\,BP = \frac{Cof\,SP\;Cof\,LB}{Cof\,SL} = \frac{Cof\,ZP\;Sin\,LS}{Cof\,LS} = Cof\,ZP\;Tg\,LS$. Patet autem valorem ipſius *BP* negativum eſſe debere, quoniam arcus $LB > 90^{\circ}$, unde ſequitur, ut declinatio Solis Auſtralis ſit, ubi crepuſculum minimum fuerit. Pro Latitudine igitur $60^{\circ}.27'.7''$ habebitur ope formulæ allatæ declinatio Solis Auſtralis in caſu crepuſculi minimi $7^{\circ}.55'.11''$, unde ſecundum formulam §. 3. eruitur crepuſculum minimum $2^{h}.27'.57''$, quod etiam a tabula ſupra allata deducere poſſumus. Ex Ephemeridibus vero Solem hanc ipſam declinationem habere conſtat circa diem I. menſis Martii & diem I. menſis Octobris, ita ut quovis anno bis crepuſculum ſit minimum.